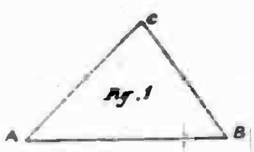
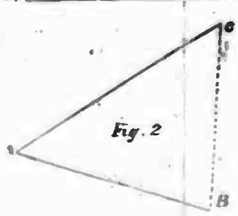
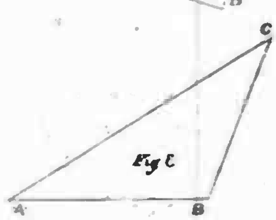
## ON THE PROPERTIES OF TRIANGLES.

TO THE EDITOR OF THE BUILDER.

SIR,—Having devoted a considerable portion of my time to the consideration and practice of the properties of triangles, I will, Sir, with your permission, endeavour through the columns of your valuable journal to explain their importance in practical geometry, and will begin by shewing their importance and utility in land surveying, navigation, and astronomy, and endeavour to give a general idea of their principles in plain trigonometry, as it relates to each of the foregoing sciences. It has been already shewn by previous theorists that all triangles consist of six parts, viz. three sides and three angles, and that when any three of these parts are given, one of which must be a side, the remaining three may be found or determined. Then in plane trigonometry it is absolutely necessary that one of the sides should be given, which can easily be shewn by drawing a diagram representing several triangles, the angles of any one of which would be exactly equal to those of the others, though the size of the triangles were all different, but the instant that the side of any one triangle is known, all the other parts may be determined; for if we suppose the side *a b* (fig. 1) and the two angles A and B to be given, it is obvious that the inclination of the lines proceeding from the point A B cannot be changed without altering the value of the given angles: the lines, if produced, must meet in one fixed and unalterable point C, and hence the lines A C and B C and the angle C may be easily determined.

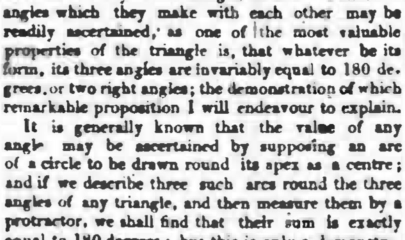


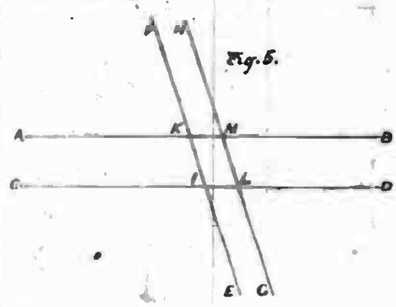




In like manner, if the lines *a c* and *a b* (fig. 2) are given, it is equally evident that the relative situations of the two lines cannot be altered, without altering the given angle, and the third side of the triangle must be the true distance between the fixed points C and B and the line, C B determines the value of the other two angles. But if, as in (fig. 3) all the three sides are given, as it is impossible the lines A B, A C, and C D, should form any other triangle, it follows, that the angles which they make with each other may be readily ascertained, as one of the most valuable properties of the triangle is, that whatever be its form, its three angles are invariably equal to 180 degrees, or two right angles; the demonstration of which remarkable proposition I will endeavour to explain.

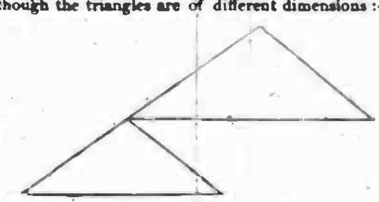
It is generally known that the value of any angle may be ascertained by supposing an arc of a circle to be drawn round its apex as a centre; and if we describe three such arcs round the three angles of any triangle, and then measure them by a protractor, we shall find that their sum is exactly equal to 180 degrees; but this is only a demonstration such as would suit practical men, and not such as mathematicians would require; and, therefore, as no one circle can be drawn round all the angles of a triangle, we must endeavour to transfer them all to one common centre; and if, when we have accomplished this, we find that the three angles exactly fill the space included by the two right angles, the demonstration will be complete.




By a careful study of the preceding diagram, in which the dotted semicircle *a b c d* is drawn round one of the angles of the triangle A B C, it is evident, therefore, that the angle A C B is introduced into the semicircle, and is measured by the arc *a b*. We have now only to introduce the other two angles, and this is effected by drawing the line C D parallel to the line A B; to shew that this is the case, it is necessary to revert to the demonstration that when two right lines cross each other, the opposite angles are equal, which will be explained in the following diagram:—




As from the properties of parallel lines it will be evident that as the lines A B and C D (fig. 5) are known to be parallel to each other, and the lines E F and G H crossing them are also parallel to each other, and the alternate angles formed by their intersection are equal, and that all angles formed on the same side of a right line, by lines drawn parallel to each other, are also equal to each other, as well as to their opposite angles; thus the alternate angles A K I and K I L, or the alternate angles K M L and M L D, are equal to each other; and it will be seen that the lines by which they are formed bear some resemblance to the letter Z, the angles F K M and H M B being formed on the same side of the line A B by the parallel lines E F and H G are also equal to each other, as well as to their opposite angles A K I and K M L; and if we imagine the parallel lines A B and C D to form but one line of considerable breadth, we shall see that the opposite angles, F K M and C I C, or the opposite angles, H M B and I L G, are also equal to each other; the same observations will apply to the alternate and opposite obtuse angles in the figure, all of which are necessarily equal. Now if we apply these principles to the triangle A B C (fig. 4), that as the line C D is parallel to the line A B, the alternate angles, A B C and B C D, must be equal to each other, and consequently the angle A B C is transferred to the semicircle *a b c d*, its value being measured by the arc *b c*, and it will be seen by following the direction of the letters A B C D, that the lines bear the same resemblance to the letter Z as in fig. 5; thus we have two angles of the triangle included within the semicircle, and is measured by its remaining side, *c d*; and as the three angles exactly fill the semicircle; it is demonstrated by the foregoing that they are equal to two right angles, or 180 degrees; then, from the foregoing demonstrations, we may conclude that the angles of all triangles which are turned the same way, and have parallel sides, are equal to each other, as in the following figure all the corresponding angles of the two triangles are respectively equal, though the triangles are of different dimensions:—



Hence the principle upon which similar figures are constructed, as illustrated by the plain table for drawings, plans, &c.: the same principle is further exemplified by that useful instrument called the pantograph, for enlarging, or more particularly for reducing drawings of any kind which may be explained by its operation, on which depends the constant parallelism of a series of rods moving freely on joints in any required direction; the instrument being placed on a sheet of paper, a pencil loaded with a small weight is placed vertically in one part of the frame, while a tracing point in another part is passed over the lines of the drawing to be reduced, and the motion thus communicated to the pencil causes it to describe lines exactly similar to those of the drawing, but differing in size according to the relative situation of the pencil and the tracer. The instrument may be so adjusted as to make the copy either larger than the drawing of the same size, or to reduce it to one-half, one-third, one-fourth, or any other required proportion.

Having thus far explained the general properties of triangles, the proportions which their angles bear to their sides, and the facility with which, when two angles are known, the third may be ascertained, it will be easy to see how extensive is the practical application of these principles; for there are few cases in which we are not able to get one or two angles, and as we can frequently obtain also one or two sides, remaining parts of the triangle can be found with the greatest ease and accuracy. And as the rays of light are known to move in right lines, through a medium uniform density, the lines which are ascertained by means of instrumental observation are more accurate than those which are determined by actual measurement, besides which they approach as nearly as possible to the mathematical definition of right lines.

But in order to make observations with correctness, it is indispensably necessary that the instruments used for that purpose should be constructed with the most scrupulous accuracy in the arcs of quadrants, sextants, &c. The degrees must necessarily be small; as the minutest error would be immensely magnified if the lines described by the rays of light were continued to a great distance; for example, if two straight rods are placed together so as to form an angle, and a careful observer to note any difference in the divergency of those rods, it would be found the difference of ⅛ of an inch in the divergency of the rods would be scarcely perceptible; but if the lines were supposed to be continued to the moon or a star, it must be evident that the slightest error near the centre would be astonishingly multiplied. It may be adduced, perhaps, that the liability to error would be diminished by graduating the arcs at a greater distance from the centre. As this opinion, however correct it may appear in theory, has, I think, many insuperable objections to its adoption in practice, I am of opinion that a radius of 30 inches is the largest that ought to be safely used in the construction of astronomical instruments. Mr. Troughton has, I understand, constructed an instrument of two feet radius for the Observatory at Greenwich, some time since, with others of larger dimensions since; but such is the liability of large instruments to derangement, that we cannot even approach them without incurring the risk of altering their figure and enlarging their divisions by the mere warmth of the breath and body. The older instruments for measuring angles extended no further than to a quadrant, but those in which the whole circle is introduced are far preferable. One reason of this is, that in the quadrant or sextant, only one index is used to point to the divisions, whereas in a circular instrument, two, three, or four indexes may be employed, each furnished with a vernier; and if all the readings are alike, they mutually confirm each other; as such was the case with instruments of this description on the ordnance survey of Ireland, to my own knowledge. But even with the most accurate instruments you will sometimes find a difference between the verniers, but this is seldom the case, and where it happens it is usual to take the average means of all the verniers, which, in a circular instrument, is equivalent to taking the average of as many successive observations by a quadrantal instrument. This kind of instrument ought to be furnished with a refracting telescope and cross wires, and as the circle itself is moveable, the angles might not only be read off in different parts at the same time, but by moving the circle the readings might be repeated on different parts of the same circle as often as might be considered essential to perfect accuracy.

But such extreme nicety is not required except in astronomical observations, or in the calculations of triangles of great dimensions, such as were used in the late trigonometrical survey of Ireland, where some of the triangles were from 50 to 70 and 100 miles a side, as these specific triangles were only observed by Colonel Cobly, Captain Henderson, Captain Portlock, and Lieutenant Downes, of the Royal Engineers. But it unfortunately happens that in navigation, where accuracy is so desirable, the motion of the ship renders the use of all those instruments which are constructed upon accurate principles of little use to the observer, which is much to be lamented; then I shall proceed to exemplify the use of the principal of those instruments (the theodolite), and to examine and explain its operation, which can be done by directing to a distant object the telescope connected with the instrument, and adjusting the cross wires till their intersection coincided with the object; then let the telescope be moved horizontally, and be placed upon another object, and the angle subtended by the two objects we accurately read off by the vernier. Then let the telescope be either elevated or depressed, and on another graduated circle the vertical angles of any object may be read off with equal facility, and the height of any object or space determined thereby. The theodolite is therefore of the greatest utility to land surveyors, in their measuring of estates, counties, parishes, &c.; but however important this instrument is, it is evident from its construction that it cannot be used at sea, and the same remark may be applied to all those instruments which